Lecture 11

Reading Material

Reading:

Chapter 8 Solymar and Walsh

HW

Solymar and Walsh Chapter 8 problems

8.1, 8.3, 8.6, 8.7, 8.8, 8.12, 8.13

Due Tues. March 10^{th}.

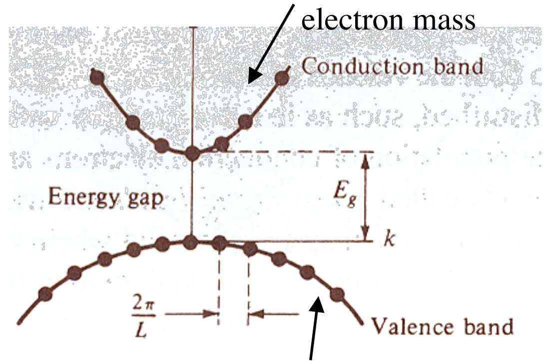
Intrinsic Semiconductors

- Now lets take $E_0 = 0$ and assume the effective masses are equal

$$m_x^* = m_y^* = m_z^* = m^*$$

$$A_x a^2 = \frac{\hbar^2}{2m_x^*},\ A_y b^2 = \frac{\hbar^2}{2m_y^*},\ A_z c^2 = \frac{\hbar^2}{2m_z^*}$$

$$E = \frac{\hbar^2}{2m^*}\left[k_x^2 + k_y^2 + k_z^2\right]$$



Energy Band Structures

- The number of states in a spherical shell (*k*-space) of thickness d*k* and radius *k* is

Number of states within dk = Number of states per unit volume of shell□dk

$$=\rho(\mathrm{k})\mathrm{d}\kappa=2\frac{4\pi \mathrm{k}^2}{\Delta V_k}\mathrm{d}\kappa = 2\frac{4\pi \mathrm{k}^2}{(2\pi)^3/V}\mathrm{d}\kappa = \frac{V\mathrm{k}^2}{\pi^2}\mathrm{d}\kappa$$

⇨ The energy of an electron with effective mass m_c and wave vector k in the conduction band is (we will assume energy depends on magnitude k)

$$E_c(\bar{k}) = \frac{\hbar^2 k^2}{2m_c}$$

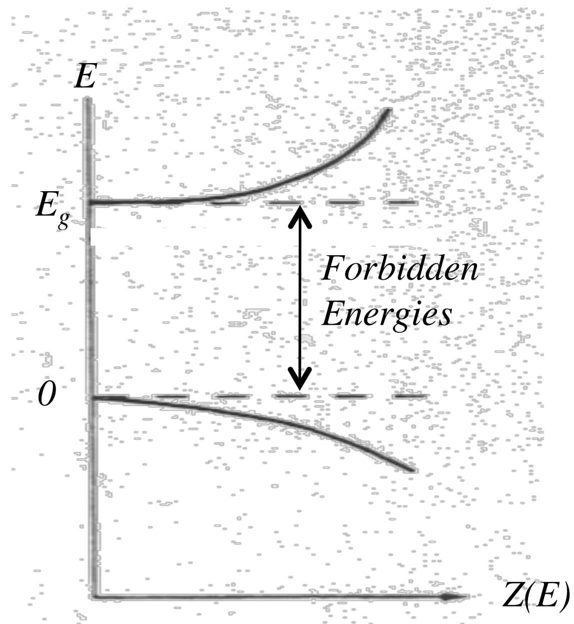
Parabola curvature determined by hole mass

Energy Band Structures

- Now let the zero energy be located at the top of the valence band, the density of states can be written as

For electrons per unit volume
$$\begin{cases} Z(E) = C_e (E - E_g)^{1/2} \\ C_e = \dfrac{4\pi\left(2m_e^*\right)^{3/2}}{\hbar^3} \end{cases}$$

For holes per unit volume
$$\begin{cases} Z(E) = C_h (0 - E)^{1/2} \\ C_e = \dfrac{4\pi\left(2m_h^*\right)^{3/2}}{\hbar^3} \end{cases}$$



Carrier Density

- The number of electron and holes per unit volume as a function of energy (N_e, N_h) are found from the number of available states per energy level per unit volume

$$N = \begin{cases} N_e = \int_{E_{c,bottom}}^{E_{c,top}} (\text{density of states})(\text{Fermi Function})\,dE, \text{ for electrons in the conduction band} \\ N_h = \int_{E_{v,top}}^{0} (\text{density of states})(\text{Fermi Function})\,dE, \text{ for holes in the valence band} \end{cases}$$

Approximation to the Fermi-Dirac Distribution

- The probability that a state at energy (E) is occupied by an electron is given by

$$f(E) = \frac{1}{e^{(E-E_F)/\kappa_B T} + 1}$$

- However, in most cases, the Fermi level sits in the the forbidden energy band, so only the tails of the distribution tend to sit in the conduction or valance band (not always, but for now we will assume this is the case).
- For this case, $E - E_F >> k_B T$ and the Fermi-Dirac tail can be approximated as an exponentially decaying function

$$F(E)\big|_{E-E_F >> \kappa_B T} \approx \exp\left[\frac{-(E-E_F)}{\kappa_B T}\right]$$

Carrier Density

- In the conduction and valance bands, the Fermi-Tails do not lend to an appreciable contribution of carriers, so we can simplify the integration by taking the energies out to $\pm\infty$.

$$N_e = \int_{E_c}^{\infty} Z(E)F(E)dE$$

$$\approx \int_{E_c = E_g|_{E_V=0}}^{\infty} C_e\left(E - E_g\right)^{1/2} \exp\left[\frac{-\left(E - E_F\right)}{\kappa_B T}\right] dE, \text{ using the substitution } x = \frac{\left(E - E_g\right)}{\kappa_B T}$$

$$= C_e \int_0^{\infty} \left(x\kappa_B T\right)^{1/2} \exp\left[\frac{-\left(x\kappa_B T + E_g - E_F\right)}{\kappa_B T}\right]\left(\kappa_B T\right)dx, \text{ since at } \mathrm{E} = \mathrm{E_g}, x = 0 \text{ and } \mathrm{E} = \infty,\ \mathrm{x} = \infty$$

$$= C_e \left(\kappa_B T\right)^{1/2} \exp\left[\frac{-\left(E_g - E_F\right)}{\kappa_B T}\right] \int_0^{\infty} (x)^{1/2} e^{-x} \left(\kappa_B T\right)dx$$

$$= C_e \left(\kappa_B T\right)^{1/2}\left(\kappa_B T\right) \exp\left[\frac{-\left(E_g - E_F\right)}{\kappa_B T}\right] \int_0^{\infty} (x)^{1/2} e^{-x} dx$$

$$= C_e \left(\kappa_B T\right)^{3/2} \exp\left[\frac{-\left(E_g - E_F\right)}{\kappa_B T}\right] \int_0^{\infty} (x)^{1/2} e^{-x} dx, \text{ using mathematical tables } \int_0^{\infty} (x)^{1/2} e^{-x} dx = \frac{\sqrt{\pi}}{2}$$

$$= C_e \frac{\sqrt{\pi}}{2} \left(\kappa_B T\right)^{3/2} \exp\left[\frac{-\left(E_g - E_F\right)}{\kappa_B T}\right]$$

$$N_e = N_c \exp\left[\frac{-\left(E_g - E_F\right)}{\kappa_B T}\right], \text{ where } N_c = C_e \frac{\sqrt{\pi}}{2}\left(\kappa_B T\right)^{3/2} = 4\pi \frac{\left(2m_e^*\right)^{3/2}}{\hbar^3} \frac{\sqrt{\pi}}{2}\left(\kappa_B T\right)^{3/2} = 2\left(\frac{2\pi m_e^* \kappa_B T}{\hbar^2}\right)^{3/2}$$

Carrier Density

- For holes, the probability of occupancy at an energy E is 1 – the probability that an electron occupies energy E, or 1-F(E). Integrating the valance band for 1-F(E) from -∞ to the top of the valance band E_v, we obtain an analogous equation for the hole density in the valance band

$$N_h = \int_{-\infty}^{E_v} Z(E)\left[1 - F(E)\right]dE$$

$$= N_v \exp\left[\frac{(0 - E_F)}{\kappa_B T}\right], \text{ where } N_v = 2\left(\frac{2\pi m_h^* \kappa_B T}{\hbar^2}\right)^{3/2}$$

Carrier Density

- In an Intrinsic semiconductor in thermal equilibrium and condition of charge neutrality, the number of electrons in the conduction band equals the number of holes in the valence band $N_e=N_h$.
- Using the condition of charge neutrality in an intrinsic semiconductor, we can solve for the Fermi level E_F

$$N_e = N_h$$

$$N_c \exp\left[\frac{-\left(E_g - E_F\right)}{\kappa_B T}\right] = N_v \exp\left[\frac{\left(0 - E_F\right)}{\kappa_B T}\right]$$

$$2\left(\frac{2\pi m_e^* \kappa_B T}{\hbar^2}\right)^{3/2} \exp\left[\frac{-\left(E_g - E_F\right)}{\kappa_B T}\right] = 2\left(\frac{2\pi m_h^* \kappa_B T}{\hbar^2}\right)^{3/2} \exp\left[\frac{\left(0 - E_F\right)}{\kappa_B T}\right]$$

$$\left(m_e^*\right)^{3/2} \exp\left[\frac{-\left(E_g - E_F\right)}{\kappa_B T}\right] = \left(m_h^*\right)^{3/2} \exp\left[\frac{-E_F}{\kappa_B T}\right]$$

$$\exp\left[\frac{2E_F}{\kappa_B T}\right]\exp\left[\frac{-E_g}{\kappa_B T}\right] = \left(\frac{m_h^*}{m_e^*}\right)^{3/2}$$

$$\frac{2E_F}{\kappa_B T} - \frac{E_g}{\kappa_B T} = \log_e\left(\frac{m_h^*}{m_e^*}\right)^{3/2}$$

$$2E_F - E_g = \frac{3}{2}\kappa_B T \log_e\left(\frac{m_h^*}{m_e^*}\right)$$

$$E_F\Big|_{inrinsic} = \frac{E_g}{2} + \frac{3}{4}\kappa_B T \log_e\left(\frac{m_h^*}{m_e^*}\right)$$